MATHEMATICS

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AIM POINTS
MATHEMATICS
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XIth, XIIth, TARGET IIT-JEE (MAIN + ADVANCE) & COMPETITIVE EXAM. FOR XII (PQRS)

DIFERENTIAL EQUATIONS

& Their Properties

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THINGS TO REMEMBER

1. $\frac{dy}{dx} = 2xy$

2. $\frac{d^2y}{dx^2} = 4x$

3. $\frac{dy}{dx} = \sin x + \cos x$

4. $\frac{dy}{dx} + 2xy + x^3$

5. $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2$

6. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{-3/2} = k\frac{d^2y}{dx^2}$

7. $y = x\frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^2}$

8. $(x^2 + y^2)\, dx - 2xy\, dy = 0$

9. $\left(\frac{d^3y}{dx^3}\right)^2 + \left(1+\frac{dy}{dx}\right)^3 = 0$

	Differential equation	Function
10.	$x\frac{dy}{dx} = 1, y(1) = 0$	$y = \log x$
11.	$\frac{dy}{dx} = y, y(0) = 1$	$y = e^x$
12.	$\frac{d^2y}{dx^2} + y = 0, y(0) = 0, y'(0) = 1$	$y = \sin x$
13.	$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0, y(0) = 2, y'(0) = 1$	$y = e^x + 1$
14.	$\frac{dy}{dx} + y = 2, y(0) = 3$	$y = e^{-x} + 2$
15.	$\frac{d^2y}{dx^2} + y = 0, y(0) = 1, y'(0) = 1$	$y = \sin x + \cos x$
16.	$\frac{d^2y}{dx^2} - y = 0, y(0) = 2, y'(0) = 0$	$y = e^x + e^{-x}$

17. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$, y (0) = 1, y' (0) 3 $\qquad y = e^x + e^{2x}$

18. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$, y (0) = 1, y' (0) 2 $\qquad y = xe^x + e^x$

EXERCISE-1

1. Determine the order and degree of each of the following differential equations. State also if they are linear or non-linear.

   (i) $\frac{\{1+(dy/dx)^2\}^{3/2}}{d^2y/dx^2} = k$ (ii) $\frac{d^2y}{dx^2} = 1 + \sqrt{\frac{dy}{dx}}$

   (iii) $y = \frac{dy}{dx} + \frac{c}{dy/dx}$ (iv) $y + \frac{dy}{dx} = \frac{1}{4}\int y\,dx$

2. In each of the following differential equations indicate its degree, wherever possible. Also, give the order of each of them.

   (i) $\frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right) = 0$ (ii) $\frac{d^5y}{dx^5} + e^{dy/dx} + y^2 = 0$

   (iii) $\frac{d^4y}{dx^4} + \sin\left(\frac{d^3y}{dx^3}\right) = 0$ (iv) $\left(\frac{d^2y}{dx^2}\right) + \cos\left(\frac{dy}{dx}\right) = 0$

3. Form the differential equation representing the family of curves y = A cos (x + B), where A and B parameters.
4. Form the differential equation of the family of curves y = a sin (bx + c), a and c being parameters.
5. Form the differential equation corresponding to $y^2 = a(b - x)(b + x)$ by eliminating parameters a and b.
6. Find the differential equation of all circles touching the
   (i) x-axis at the origin
   (ii) y-axis at the origin
7. Obtain the differential equation of all circles of radius r.
8. Find the differential equation of all the circles in the first quadrant which touch the coardinate axes.
9. Find the differential equation of family of paarbolas having vertex at the origin and axis along positive y-axis.
10. Find the differential equation of the family of ellipses having foct on y-axis and centre the origin.
11. Show that the differential equation representing one parameter family of curves $(x^2 - y^2) = c(x^2 + y^2)^2$ is $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$.
12. Represent the following family of curves by forming the corresponding differential equations (a, b are parameters) :

    (i) $\frac{x}{a} + \frac{y}{b} = 1$ (ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

    (iii) $(y - b)^2 = 4(x - a)$
13. Find the differential equation of the family of curves, x = A cos nt + B sin nt, where A and B are arbitrary constants.
14. Form the differential equation corresponsing to $y^2 - 2ay + x^2 = a^2$ by eliminating a.

15. Form the differential equation of the family of curves represented by the equation (a being the parameter) :
    (i) $(2x + a)^2 + y^2 = a^2$
    (ii) $(2x - a)^2 - y^2 = a^2$
    (iii) $(x - a)^2 + 2y^2 = a^2$
16. Represent the following families of curves by forming the corresponding differential equations (a, b being parameters) :
    (i) $x^2 + y^2 = a^2$ (ii) $x^2 - y^2 = a^2$
    (iii) $y^2 = 4ax$ (iv) $x^2 + (y - b)^2 = 1$
    (v) $(x - a)^2 - y^2 = 1$ (vi) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
    (vii) $y^2 = 4a(x - b)$ (viii) $y = ax^3$
    (ix) $x^2 + y^2 = ax^3$ (x) $y = e^{ax}$
17. Form the differential equation representing the family of ellipses having centre at the origin and foci on x-axis.
18. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at the origin.
19. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.
20. Show that $xy = ae^x + be^{-x} + x^2$ is a solution of the differential equation $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0$.
21. Verify that the function $y = C_1 e^{ax} \cos bx + C_2 e^{ax} \sin bx$, C_1, C_2 are arbitrary constants is a solution of the differential equation $x\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 + b^2) y = 0$.
22. Show that the function $y = A \cos x + B \sin x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$
23. Show that the function $y = A \cos 2x - B \sin 2x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$
24. Show that $y = e^x (A \cos x + B \sin x)$ is the solution of the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$
25. Verify that $y^2 = 4a(x + a)$ is a solution of the differential equations $y\left\{1 - \left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$
26. Verify that $y = ce^{\tan^{-1}x}$ is a solution of the differential equations $(1 + x^2)\frac{d^2y}{dx^2} + (2x - 1)\frac{dy}{dx} = 0$
27. Verify that $y\,e^{m\cos^{-1}x}$ satisfies the differential equation $(1 - x)^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$
28. Verify that $y = \log(x + \sqrt{x^2 + a^2})^2$ satisfies the differential equation $(a^2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0$

29. Show that $y = e^{-x} + as + b$ is solution of the differential equation $e^x \frac{d^2y}{dx^2} = 1$

30. For each of the following difereential equations verify that the accompanying function is a solution in the mentioned domain (a, b are parameters).

Differential equation	Function
(i) $x\frac{dy}{dx} = y$	$y = ax, x \in R - \{0\}$
(ii) $x + y\frac{dy}{dx} = 0; x \in R, y \neq 0$	$y = \pm\sqrt{a^2 - x^2}, x \in (-a, a)$
(iii) $x\frac{dy}{dx} + y = y^2; x \in R - \{0\}$	$y = \frac{a}{x+a}, x \in R - \{a\}$
(iv) $x^2\frac{d^2y}{dx^2} = 1, x \in R - \{0\}$	$y = ax + b + \frac{1}{2x}, x \in R - \{0\}$
(v) $y = \left(\frac{dy}{dx}\right)^2, x \in R, y \geq 0$	$y = \frac{1}{4}(x \pm a)^2, x \in R$

31. Verify that the function defined by $y = \sin x - \cos x, x \in R$ is a solution of the initial value problem $\frac{dy}{dx} = \sin x + \cos x, y(0) = -1$.

32. Solve :

(i) $\frac{dy}{dx} = \frac{x}{x^2+1}$ (ii) $(e^x + e^{-x})\frac{x}{x^2+1} = (e^x - e^{-x})$

33. Solve the following differential equation $\frac{dy}{dx} = x^2 + x - \frac{1}{x}, x \neq 0$

34. Solve the following differential equation $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

35. Solve the following differential equation $\frac{dy}{dx} = \tan^{-1} x$

36. Solve the following differential equation $\frac{1}{x}\frac{dy}{dx} = \tan^{-1} x, x \neq 0$

37. Solve the following differential equation $(\sin x + \cos x)\, dy + (\cos x - \sin x)\, dx = 0, x \neq n\pi + \frac{3\pi}{4}$, $n \in Z$

38. Solve the following differential equation $\frac{dy}{dx} - x \sin^2 x = \frac{1}{x \log x}$

39. Solve the following differential equation $\sqrt{1-x^4}\, dy = x\, dx$

40. Solve the following differential equation $(1 + x^2)\frac{dy}{dx} - x = 2\tan^{-1}x$

41. Solve the following differential equation $(x^3 + x^2 + x + 1)\frac{dy}{dx} = 2x^2 + x$

42. Solve :

(i) $\frac{dy}{dx} = \frac{1}{y^2 + \sin y}, y \neq 0$

(ii) $\frac{dy}{dx} = \sec y, y \neq (2n + 1)\frac{\pi}{2}, n \in Z$

43. Solve : $\frac{dy}{dx} + y = 1$

44. Solve the initial value problem $\frac{dy}{dx} + 2y^2 = 0, y(1) = 1$ and find the corresponding solution curve.

45. Solve :

(i) $(x + 1)\frac{dy}{dx} = 2xy$

(ii) $\cos x (1 + \cos y)\, dx - \sin y (1 + \sin x) dy = 0$

46. Solve :

(i) $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

(ii) $e^x\sqrt{1 - y^2}\, dx + \frac{y}{x} dy = 0$

47. Solve the differential equation $(1 + e^{2x})dy + (1 + y^2) e^x dx = 0$ given that when $x = 0, y = 1$.

48. Solve :

(i) $(x^2 - yx^2)\, dy + (y^2 + x^2y^2)\, dx = 0$

(ii) $3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$

49. Solve :

(i) $\sin^3 x \frac{dx}{dy} = \sin y$

(ii) $\frac{dy}{dx} + \sqrt{\frac{1 - y^2}{1 - x^2}} = 0$

50. Solve :

(i) $\frac{dy}{dx} = e^{x - y} + x^2 e^{-y}$

(ii) $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$

51. Solve $\frac{dy}{dx} = y \sin 2x$, it being given that $y(0) = 1$.

52. Find the equation of the curve passing through the point $\left(0, \frac{\pi}{4}\right)$ whose differential equation is $\sin x \cos y\, dx + \cos x \sin y\, dy = 0$.

53. Solve the following initial value problems :

(i) $(x + 1)\frac{dy}{dx} = 2e^{-y} - 1$, y(0) = 0

(ii) $y - x\frac{dy}{dx} = 2\left(1+x^2\frac{dy}{dx}\right)$, y(1) = 1

54. Show that the general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1} = 0$ is given by x + y + 1 = A(1 – x – y – 2xy), where A is a parameter.

55. Find the particular solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ given that y = 0 when x = 0.

56. In a bank principal increases at the rate of 5% per year. In how many years Rs. 1000 double itself.

57. Find teh equation of the curve passing through the point (0, –2) given that at any point (x, y) on the curve the product of the slope of its tangent and y coordinate of the point is equal to the x-coordinate of the point.

58. At any point (x, y) of a curve the slope of the tangent it twice the slope of the line segment joining the point of contact to the point (–4, –3). Find the equation of the curve given that it passes through (–2, 1).

59. Slove the following differential equation $(x - 1)\frac{dy}{dx} = 2xy$

60. Slove the following differential equation $(1 + x^2)\,dy = xy\,dx$

61. Slove the following differential equation $x \cos y\,dy = (x e^x \log x + e^x)\,dx$

62. Slove the following differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

63. Slove the following differential equation $\frac{dy}{dx} = \frac{e^x(\sin^2 x+\sin 2x)}{y(2\log y+1)}$

64. Slove the following differential equation $\frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y+y\cos y}$

65. Slove the following differential equation $\frac{dy}{dx}+\frac{\cos x\sin y}{\cos y} = 0$

66. Slove the following differential equation $(y + xy)dx + (x - xy)^2dy = 0$

67. Slove the following differential equation $(xy^2 + 2x)\,dx + (x^2y + 2y)\,dy = 0$

68. Slove the following differential equation $y(1 - x^2)\frac{dy}{dx} = x(1 + y^2)$

69. Slove the following differential equation $\frac{dy}{dx} = y \tan x$, y (0) = 1

70. Slove the following differential equation $2x\frac{dy}{dx} = 5y$, y(1) = 1

71. Slove the following differential equation $\frac{dy}{dx} = 2e^{2x} y^2$, $y(0) = -1$

72. Slove the following differential equation $\cos y \frac{dy}{dx} = e^x$, $y(0) = \frac{\pi}{2}$

73. Slove the following differential equation $\frac{dy}{dx} = 2xy$, $y(0) = 1$

74. Solve the differential equation $x\frac{dy}{dx} + \cot y = 0$, given that $y = \frac{\pi}{4}$ when $x = \sqrt{2}$.

75. Solve the differential equation $(1 + x^2)\frac{dy}{dx} + (1 + y^2) = 0$, given that $y = 1$ when $x = 0$.

76. Solve the differential equation $\frac{dy}{dx} = \frac{2x(\log x + 1)}{\sin y + y \cos y}$, given that $y = 0$, when $x = 1$.

77. Find the equation of a curve passing through the point (0, 0) and whose differential equation is $\frac{dy}{dx} = e^x \sin x$.

78. For the differential equation $xy\frac{dy}{dx} = (x + 2)(y + 2)$. Find the solution curve passing through the point (1, –1).

79. The volume of a spherical ballon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of the balloon after t second.

80. In a bank principal increases at the rate of r% per year. Find the value of r if Rs. 100 double itself in 10 years ($\log_e 2 = 0.6931$).

81. In a back principal inreases at the rate of 5% per year. An amount of Rs. 100 is deposited with this bank, how much will it worth after 10 years ($e^{0.5} = 1.648$).

82. In a culture the bacteria count is 100000. The number is increased by 10% in 2 hours. In hiw many hours will the count reach 200000, if the rate of growth of bacteria is proportional to the number present.

83. Solve :

    (i) $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

    (ii) $\frac{dy}{dx} = (4x + y + 1)^2$

84. Solve : $(x + y)^2 \frac{dy}{dx} = a^2$

85. Solve the initial value problem : $\cos(x + y)\, dy = dx$, $y(0) = 0$.

86. Solve the following initial value problems :

    (i) $(x + y + 1)^2\, dy = dx$, $y(-1) = 0$

    (ii) $(x - y)(dx + dy) = dx - dy$, $y(0) = -1$

87. Solve the following differential equations $\frac{dy}{dx} = (x + y + 1)^2$

88. Solve the following differential equations $\frac{dy}{dx}.\cos(x - y) = 1$

89. Solve the following differential equations $\frac{dy}{dx} = \frac{(x-y)+3}{2(x-y)+5}$

90. Solve the following differential equations $\frac{dy}{dx} = (x + y)^2$

91. Solve the following differential equations $\cos^2(x - 2y) = 1 - 2\frac{dy}{dx}$

92. Solve the following differential equations $\frac{dy}{dx} = \tan(x + y)$

93. Solve the differential equation $x^2\,dy + y(x + y)\,dx = 0$, given that y = 1 when x = 1.
94. Solve the differential equation $(x + y)\,dy + (x - y)\,dx = 0$, given that y = 1 when x = 1.
95. Solve the differential equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$; given that y = 1 when x = 1.
96. Solve : $x^2 y\,dx - (x^3 + y^3)\,dy = 0$
97. Solve : $(x^2 + xy)\,dy = (x^2 + y^2)\,dx$.
98. Solve : $(3xy + y^2)\,dx + (x^2 + xy)\,dy = 0$.
99. Solve : $(x^3 - 3xy^2)\,dx = (y^3 - 3x^2y)\,dy$.
100. Solve : $x\,dy - y\,dx = \sqrt{x^2 + y^2}\,dx$

101. Solve : $y\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\}dx - x\left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\}dy = 0.$

102. Solve : $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$

103. Solve : $2y\,e^{x/y}\,dx + (y - 3x\,e^{x/y})dy = 0$.
104. Solve each of the following initial value problems :

(i) $2x^2\frac{dy}{dx} - 2xy + y^2 = 0,\ y(e) = e$

(ii) $2xy + y^2 - 2x^2\frac{dy}{dx} = 0,\ y(1) = 2$.

105. Solve each of the following initial value problems :
(i) $(x^2 + y^2)\,dx + xy\,dy = 0,\ y(1) = 1$
(ii) $(xe^{y/x} + y)\,dx = x\,dy,\ y(1) = 1$
(iii) $(x^2 - y^2)\,dx + 2xy\,dy = 0,\ y(1) = 1$
106. Solve the following differential equation $x^2\,dy + y(x + y)\,dx = 0$

107. Solve the following differential equation $x\frac{dy}{dx} = \frac{y-x}{y+x}$

108. Solve the following differential equation $x\frac{dy}{dx} = x + y, x \neq 0$

109. Solve the following differential equation $\frac{dy}{dx} = \frac{y+x}{y-x}$

110. Solve the following differential equation $x^2\frac{dy}{dx} = x^2 - 2y^2 + xy$

111. Solve the following differential equation $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$

112. Solve the following differential equation $xy \log\left(\frac{x}{y}\right)dx + \left\{y^2 - x^2 \log\left(\frac{x}{y}\right)\right\}dy = 0$

113. Solve the following differential equation $(x - y)\frac{dy}{dx} = x + 2y$

114. Solve the following differential equation $(2x^2 y + y^3)dx + (xy^2 - 3x^3) dy = 0$

115. Solve the following differential equation $x\frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$

116. Solve the following differential equation $y\, dx + \left\{x \log\left(\frac{y}{x}\right)\right\} dy - 2x\, dy = 0$

117. Solve each of the following initial value problems :
   (i) $(x^2 + y^2)\, dx = 2xy\, dy,\ y(1) = 0$
   (ii) $xe^{y/x} - y + x \sin\left(\frac{y}{x}\right) = 0$
   (iii) $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\frac{y}{x} = 0,\ y(1) = 0$
   (iv) $(y^4 - 2x^3 y)\, dx + (x^4 - 2xy^3)dy = 0,\ y(1) = 0$

118. Solve the differential equation $\frac{dy}{dx} - \frac{y}{x} = 2x^2, x > 0$

119. Solve the differential equation $x \log x\frac{dy}{dx} + y = \frac{2}{x}\log x, x > 0$

120. Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$

121. Solve : $\frac{dy}{dx} + y \sec x = \tan x\left(0 \leq x < \frac{\pi}{2}\right)$

122. Solve : $\cos^2 x\frac{dy}{dx} + y = \tan x\left(0 \leq x < \frac{\pi}{2}\right)$

123. Solve : $(x^2 + 1)\frac{dy}{dx} + 2xy = \sqrt{x^2 + 4}$

124. Solve : $(1 + x^2)\frac{dy}{dx} = 2xy - 4x^2 = 0$ subject to the initial condition $y(0) = 0$.

125. Solve each of the following initial value problems :

(i) $\frac{dy}{dx} + \frac{2x}{x^2+1}y = \frac{1}{(x^2+1)^2}$, $y(0) = 0$

(ii) $(x^2 + 1)\, y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 0$

126. Solve : $x\frac{dy}{dx} + y - x + xy\cot x = 0$, $x \neq 0$

127. Solve : $y\,dx - (x + 2y^2)\,dy = 0$

128. Solve : $y\,dx + (x - y^3)\,dy = 0$

129. Solve : $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dy}{dx} = 1$, $x \neq 0$

130. Solve each of the following initial value problems :

(i) $(x - \sin y)\,dy + (\tan y)\,dx = 0$, $y(0) = 0$

(ii) $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$, $y(0) = 0$

131. Solve the following differential equation $\frac{dy}{dx} + 2y = e^{3x}$

132. $4\frac{dy}{dx} + 8y = 5\,e^{-3x}$

133. $\frac{dy}{dx} + 2y = 6\,e^{x}$

134. $x\frac{dy}{dx} + y = x\,e^{x}$, $x > 0$

135. $\frac{dy}{dx} + \frac{4x}{x^2+1}y + \frac{1}{(x^2+1)^2} = 0$

136. $x\frac{dy}{dx} + y = x\log x$

137. $x\frac{dy}{dx} - y = (x - 1)\,e^{x}$

138. $\frac{dy}{dx} = y\tan x - 2\sin x$

139. $(1 + x^2)\frac{dy}{dx} + y = \tan^{-1} x$

140. $\frac{dy}{dx} + y\tan x = \cos x$

141. $\frac{dy}{dx} = y \cot x = x^2 \cot x + 2x$

142. $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

143. $(1 + y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

144. $(1 + x^2)\frac{dy}{dx} - 2xy = (x^2 + 2)(x^2 + 1)$

145. $\frac{dy}{dx} - y = xe^x$

146. $\frac{dy}{dx} + 2y = xe^{4x}$

147. Find one-parameter families of solution curves of the following differential equations : (or Solve the following differential equations)

(i) $x \log x \frac{dy}{dx} + y = 2 \log x$

(ii) $x\frac{dy}{dx} + 2y = x^2 \log x$

148. Solve each of the following initial value problems :

(i) $\frac{dy}{dx} + y = x \cos x + \sin x,\ y\left(\frac{\pi}{2}\right) = 1$

(ii) $\frac{dy}{dx} + 2y \tan x = \sin x;\ y = 0$ when $x = \frac{\pi}{3}$

(iii) $\frac{dy}{dx} - 3y \cot x = \sin 2x;\ y = 2$ when $x = \frac{\pi}{2}$

149. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2\ (x \neq 0)$

150. Find the general solution of thedifferential equation $\frac{dy}{dx} - y = \cos x$

151. The surface area of a balloon eing inflated changes at a constant rate. If initially, its radius is 3 units and after 2 seconds, it is 5 units, find the radius after t seconds.

152. The temperature T of a cooling object drops at a rate proportional t the difference T - S, where S is constant temperature of surrounding medium. If initially T = 150°C, find the temperature of the cooling object at any time t.

153. Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature 25°C. Find

(i) the temperature of water after 20 minutes.

(ii) The time when the temperature is 40°C. [Given : $\log_e \frac{11}{15} = -0.3101$, log 5 = 1.6094]

154. A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50°F. What is the temperature outside ?

155. The doctor took the temperature of a dead body at 11.30 PM which was 94.6°F. He took the temperature of the body again after one hour, which was 93.4°F. If the temperature of the room was 70°F, estimate the time of death. Taking normal temperature of human body as 98.6°F.

$$\left[\text{Given}: \log\frac{143}{123} = 0.15066, \log\frac{123}{117} = 0.05\right]$$

156. The slope of the tangent to the curve at any point is twice the ordinate at that point. The curve passes through the point (4, 3). Determine its equation.

157. The normal lines to a given curve at each point pass through (2, 0). The curve passes through (2, 3). Formulate the differential equation and hence find out the equation of the curve.

158. The slope of the tangent to a curve at any point (x, y) on it is given by $\frac{y}{x} - \cot\frac{y}{x}.\cos\frac{y}{x}, (x > 0, y > 0)$ and the curve passes through the point $\left(1, \frac{\pi}{4}\right)$. Find the equation of the curve.

159. Show that the curve for which the portion of the tangent at any point of it included between the coordinate axes is bisected at the point of contact is a rectangular hyperbola xy = C.

160. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2 + y^2}{2xy}$ is given by $x^2 - y^2 = kx$.

161. Experiments show that the rate of inversion of cane-sugar in dilute solution is proportional to the concentration y(t) of the unaltered solution. Suppose that the concentration is $\frac{1}{100}$ at t = 0 and $\frac{1}{300}$ at t = 10 hours. Find y(t).

162. The surface area of a ballon being inflated,changes at a rate proportional to time t. If initially its radius is 1 unit and after 3 seconds it is 2 units, find the radius after time t.

163. The rate of growth of a population is proportional to the number present. If the population of a city doubled in the past 25 years, and the present population is 100000, when will the city have a population of 500000 ? [Given $\log_e 5 = 1.609$, $\log_e 2 = 0.6931$.]

164. The population of a city increases at a rate proportional to the number of inhabitants present at any time t. If the population of the city was 200000 in 1990 and 250000 in 2000, what will be the population in 2010 ?

165. If the marginal cost of manufacturing a certain item is given by $C(x) = \frac{dC}{dx} = 2 + 0.15x$.

Find the total function C(x), given that C(0) = 100.

166. Find the equation of the curve which passes through the point (2, 2) and satisfies the differential equation $y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$.

167. Show that the equation of the curve whose slope at any points is equal to y + 2x and which passes through the origin is $y + 2(x + 1) = 2e^{2x}$.

168. Find the equation of the curve such that the portion of the x-axis cut off between the origin and the tangent at a point is twice the bascissa and which passes through the point (1, 2).

169. Find the equation to the curve satisfying $x(x + 1)\frac{dy}{dx} - y = x(x + 1)$ and passing through (1, 0).

170. The normal to a given curve at each point (x, y) on the curve passes through the point (3, 0). If the curve contains the point (3, 4), find its equation.

171. Radium decomposes at a rate proportional to the quantity of radium present. It is found that in 25 years, approximately 1.1% of a certain quantity of radium has decomposed. Determine approximately how long it will take for one-half of the orginal amount of radium to decompose ?

[Given $\log_e 0.989 = 0.01106$ and $\log_e 2 = 0.6931$]

172. Find the equation of the curve passing through the point (0, 1) if the slope of the tangent to the curve at each of its point is equal to the sum of the abscissa and the product of the abscissa and the ordinate of the point.

173. The slope of a curve at each of its points is equal to the square of the abscissae of the point. Find the particular curve through the point (-1, 1).

EXERCISE-2

1. Write the degree of the different equation $a^2\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/4}$

2. Write the order of the different equation $1 + \left(\frac{dy}{dx}\right)^2 = 7\left(\frac{d^2y}{dx^2}\right)^3$

3. Write the order and degree of the different equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

4. Write the degree of the different equation $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2x^2 \log\left(\frac{d^2y}{dx^2}\right)$

5. If sin x is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$, then write the value of P.

6. What is the degree of the following differential equation ?

$$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log^x$$

7. Determine the order and degree (if defined) of the following different equations :

(i) $\left(\frac{dx}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$

(ii) $y''' + 2y'' + y' = 0$

(iii) $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

(iv) $y''' + 2y'' + y' = 0$

(v) $y'' + (y')^2 + 2y = 0$

8. Verify the the function $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

9. In each of the following verify that the given function (explicit of implicit) is a solution of the corresponding differential equation :

(i)	$y = e^x + 1$	$y'' - y' = 0$
(ii)	$y = x^2 + 2x + C$	$y' - 2x - 2 = 0$
(iii)	$y = \cos x + C$	$y' + \sin x = 0$
(iv)	$y = \sqrt{1+x^2}$	$y' = \frac{xy}{1+x^2}$
(v)	$y = x \sin x$	$xy' = y + x\sqrt{x^2 - y^2}$ $(x \neq 0)$ and $x > y$ or $x < -y$)
(vi)	$y = \sqrt{a^2 - x^2}$, $x \in (-a, a)$	$x + y\frac{dy}{dx} = 0$, $y \neq 0$

10. Form the differential equation representing the family or curves $y = a \sin(x + b)$, where a, b are arbitrary constant.

11. Form the differential equation representing the family of parabolas having vertex at origin and axis along positive direction of x-axis.

12. From the differential equation of the family of hyperbolas having foci on x-axis and centre at the origin.

13. Verify that $xy = a e^x + b e^{-x} + x^2$ is a solution of the differential equation $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy + x^2 - 2 = 0$

14. Show that $y = Cx + 2C^2$ is a solution of the differential equation $2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$

15. Show that $y^2 - x^2 - xy = a$ is a solution of the differential equation $(2 - 2y)\frac{dy}{dx} + 2x + y = 0$

16. Verify that $y = A \cos x + \sin x$ satisfies the differential equation $\cos x \frac{dy}{dx} + (\sin x) y = 1$.

17. Show that the differential equation of all parabolas which have their axes parallel to y-axis is $\frac{d^3y}{dx^3} = 0$.

18. $(1 + x) y\, dx + (1 + y) x\, dy = 0$

19. $\cos y \log (\sec x + \tan x)\, dx = \cos x \log (\sec y + \tan y)\, dy$
20. $\operatorname{cosec} x (\log y)\, dy + x^2y\, dx = 0$
21. $\frac{dy}{dx} = \frac{\sin x + x\cos x}{y(2\log y + 1)}$
22. $y - x\frac{dy}{dx} = b\left(1 + x^2\frac{dy}{dx}\right)$
23. For each of the following differential equation, find the general solution :
    (i) $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$
    (ii) $\frac{dy}{dx} = \sqrt{4-y^2}$, $-2 < y < 2$
    (iii) $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$
    (iv) $y \log y\, dx - x\, dy = 0$
    (v) $\frac{dy}{dx} + y = 1$, $y \neq 1$
24. For each of the following differential equations, find a particulars solution satisfying the given condition :
    (i) $x(x^2 - 1)\frac{dy}{dx} = 1$, $y = 0$ when $x = 2$
    (ii) $\cos\left(\frac{dy}{dx}\right) = a$, $y = 1$ when $x = 0$
    (iii) $\frac{dy}{dx} = y \tan x$, $y = 1$ when $x = 0$
25. Solve the each of the following differential equations :
    (i) $(x - y)\frac{dy}{dx} = x + 2y$
    (ii) $y\, dx + x \log\left(\frac{y}{x}\right) dy - 2x\, dy = 0$
    (iii) $\frac{dy}{dx} + 3y = e^{-2x}$
    (iv) $\frac{dy}{dx} + (\sec x)\, y = \tan x \left(0 < x < \frac{\pi}{2}\right)$
    (v) $x\frac{dy}{dx} + 2y = x^2 \log x$
    (vi) $(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$

(vii) $(x + y)\frac{dy}{dx} = 1$

(viii) $y\,dx + (x - y^2)\,dy = 0$

26. Find a particular solution of each of the following differential equations :

$(x + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; y = 0, where x = 1

27. Find the equation of the curve passing through the point (1,1) whose differential equation is $x\,dy = (2x^2 + 1)\,dx$, $x \neq 0$.

28. Find the equation of a curve passing through the point (–2, 3), given that the slope 2r of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$.

29. Find the equation of a curve gassing through the point (0,0) and whose differential equation is $\frac{dy}{dx} = e^x \sin x$.

30. At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (–4, –3). Find the equation of the en that it passes through (–2, 1).

31. Show that the family of curves for which the slope of the tangent at any point (x, y) on it is $\frac{x^2 + y^2}{2xy}$ is given by $x^2 - y^2 = Cx$.

32. Find the equation of a curve passing throught the point (0, 1). If the slope of the tangent to the curve at any point (x, y) is equal to the sum of the x-co-ordinate and the product of the x-co-ordinate and y-co-ordinate of that point.

33. Find the equation of the curve passing through the origin given that the slope of the tangent to the curve at any point (x, y) is equal to the sum of the co-ordinates of the point.

34. Find the equation of the curve passing through the point (0,2) given that the sum of the co-ordinates of any point on the curve exceeds the magnitude of the slope of the tangent to the curve at that point by 5.

35. The slope of the tangent to the curve at any point is the reciprocal of twice the ordinate at that point The curve passes through the point (4, 3). Determine its equation.

36. The decay rate of radium at any tinte t is proportional to its mass at that time. Find f the^ime when the mass will be halved of its initial mass.

37. Experiments show that radium disintegrates at a rate proportional to the amount of radium present at the moment. Its half-life is 1590 years. What percentage will disappear in one year ?

[Use $e^{-\frac{\log 2}{1590}} = 0.9996$]

38. A wet porous substance in the open air loeses its moisture at a rate proportional to / the moisture content. If a sheet hung in the wind loses half of its moisture during the first hour, when will it have lost 95% moisture, wheather conditions remaining the same.

EXERCISE-3

1. The differential equation obtained on eliminating A and B from y = A cos ω t = B sin ω t, is
(a) $y'' + y' = 0$ (b) $y'' - \omega^2 y = 0$ (c) $y'' = -\omega^2 y$ (d) $y'' + y = 0$

2. The solution of the different equation $\frac{dy}{dx} = \frac{ax+g}{by+f}$ represent a circle when
(a) a = b (b) a = –b (c) a = –2b (d) a = 2b

3. The order of the differential equation satisfying $\sqrt{1-x^4}a + \sqrt{1-y^4} = a(x^2 - y^2)$ is
(a) 1 (b) 2 (c) 3 (d) 4

4. The differential equation of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = C$ is
(a) $\frac{y''}{y'} + \frac{y'}{y} - \frac{1}{x} = 0$ (b) $\frac{y''}{y'} + \frac{y'}{y} + \frac{1}{x} = 0$ (c) $\frac{y''}{y'} - \frac{y'}{y} - \frac{1}{x} = 0$ (d) none of these

5. Solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is
(a) x (y + cos x) = sin x + C (b) x (y – cos x) = sin x + C
(c) x (y + cos x) = co x + C (d) none of these

6. The solution of the differential equation $2x\frac{dy}{dx} = y + x\tan\frac{y}{x}$, is
(a) $\sin\frac{x}{y} = x + C$ (b) $\sin\frac{y}{x} = Cx$ (c) $\sin\frac{x}{y} = Cy$ (d) $\sin\frac{y}{x} = Cy$

7. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is
(a) $\phi\left(\frac{y}{x}\right) = kx$ (b) $x\phi\left(\frac{y}{x}\right) = k$ (c) $\phi\left(\frac{y}{x}\right) = ky$ (d) $y\phi\left(\frac{y}{x}\right) = k$

8. The solution of the differential equation $x\,dx + y\,dy = x^2 y\,dy - y^2 x\,dx$, is
(a) $x^2 - 1 = C(1 + y^2)$ (b) $x^2 + 1 = C(1 - y^2)$
(c) $x^3 - 1 = C(1 + y^3)$ (d) $x^3 + 1 = C(1 - y^3)$

9. The solution of the differential equation $(1 + x^2)\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$, is
(a) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} C$ (b) $\tan^{-1} y - \tan^{-1} x = \tan^{-1} C$
(c) $\tan^{-1} y \pm \tan^{-1} x = \tan C$ (d) $\tan^{-1} y + \tan^{-1} x = \tan^{-1} C$

10. What is integrating factor of $\frac{dy}{dx} + y\sec x = \tan x$?
(a) sec x + tan x (b) log (sec x + tan x) (c) $e^{\sec x}$ (d) sec x

11. Which of the following differential eauations has $y = C_1 e^x + C_2 e^{-x}$ as the general solution ?
(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{d^2y}{dx^2} + 1 = 0$ (d) $\frac{d^2y}{dx^2} - 1 = 0$

12. Which of the following is a homogeneous differential equation ?
(a) $4x + 6y + 5) dy - (3y + 2x + 4) dx = 0$
(b) $xy\, dx - (x^3 + y^3) dy = 0$
(c) $(x^3 + 2y^2) dx + 2xy\, dy = 0$
(d) $y^2 dx + (x^2 - xy - y^2) dy = 0$

13. The general solution of a differential equation of the type $\frac{dy}{dx} + P_1 x = Q_1$ is

(a) $y e^{\int P_1 dy} = \int \left\{ Q_1 e^{\int P_1 dy} \right\} dy + C$

(b) $y e^{\int P_1 dx} = \int \left\{ Q_1 e^{\int P_1 dx} \right\} dy + C$

(c) $x e^{\int P_1 dy} = \int \left\{ Q_1 e^{\int P_1 dy} \right\} dy + C$

(d) $x e^{\int P_1 dx} = \int \left\{ Q_1 e^{\int P_1 dx} \right\} dx + C$

14. The general solution of the differential equation $e^x dy + (y e^x + 2x) dx = 0$ is
(a) $x e^y + x^2 = C$ (b) $x e^y + y^2 = C$ (c) $y e^x = x^2 = C$ (d) $y e^y + x^2 = C$